P-1.1 & P-1.2

PEDAGOGY OF MATHEMATICS

Course Content:

Unit I: Nature and Scope of Mathematics

a. Concept of Mathematics: Meaning, nature (Truth, logic, reasoning, mathematical language & symbolism) and building blocks of Mathematics (Axioms, Propositions, Postulates, Quantifiers)

b. Mathematical propositions-Types (truth values, truth tables, Open sentences, logically valid conclusions, implications - necessary and sufficient conditions) and Proofs (direct, converse, inverse and contrapositive)

c. Contribution of mathematicians - Aryabhatta, Ramanujan, Pythagoras & Euclid; Aesthetics by Birkhoff.

Unit II: Aims, Objectives and Approaches to Teaching School Mathematics

a. Need for establishing general objectives for teaching mathematics; Study of the aims and general objectives of teaching mathematics vis-à-vis the objectives of school education;

b. Writing specific objectives and teaching points of various content areas in mathematics like Algebra, Geometry, Trigonometry with special reference to Bloom's Taxonomy

c. Approaches - Activity based, Inductive- Deductive, Analytic-synthetic and Problem Solving.

UNIT III: Learning Resources in Mathematics

a. Textbooks- need, importance, quality,

b. Audio-visual multimedia–Selection and designing;

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c. Using community resources for mathematics learning, pooling of learning resources in school complex/block/district level, handling hurdles in utilising resources.

UNIT IV: Current Trends in Teaching and Learning Mathematics

a. Concepts: Meaning, nature, concept formation and concept assimilation; Concept Attainment Model in teaching mathematics

b. Cooperative Learning: concept and approaches

Supplementary text material, summer programmes, correspondence course

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UNIT I: NATURE AND SCOPE OF MATHEMATICS

MEANING OF MATHEMATICS –

1. "Education should be started with mathematics. For it forms well designed brains that are able to reason right. It is even admitted that those who have studied mathematics during their childhood should be trusted, for they have acquired solid bases for arguing which become to them a sort of second nature". Ibn Khaldun, al-Muqaddima (born in 1332, Tunis), historian, sociologist, philosopher Strongest personalities of Arabo-Muslim culture in the period of its deline.

2. "Dieu a utilise de merveilleuses mathematiques en creant le monde"

MATHEMATICS

1**. The definition of mathematics from Britannica concise encyclopedia** is " Science of structure, order, and relation that has evolved from counting, measuring and describing the shapes of object. It deals with logical reasoning and quantitative calculations."

2. **The literal meaning of mathematics** is "things which can be counted" now you can think that counting has vital role in our daily life; just imagine that there were no mathematics at all, how would it be possible for us to count days, months and years?

DEFINATIONS OF MATHEMATICS:

"Mathematics in its widest sense is the development of all types of deductive reasoning." **-WHITEHEAD**

"Mathematics is the language of physical sciences and certainly no more marvellous language was ever created by the mind of man."**-LINDSAY**

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"Mathematics is a way to settle in the mind a habit of reasoning." **-LOCKE**

According to various definitions, mathematics is the science of measurement, quantity and magnitude. According to *New English Dictionary* "Mathematics in a strict sense is the abstract science which investigates deductively the conclusions implicit in the elementary conceptions of spatial and numerical relations." It has also been defined as the science of number and space. Its Hindi or Punjabi name is 'Ganita' which means the science of calculation. It is a systematised, organised and exact branch of science.

The term 'Mathematics' has been interpreted and explained in various ways. It is the numerical and calculation part of man's life and knowledge. It helps the man to give exact interpretation to his ideas and conclusions. It deals with quantitative facts and relationships as well as with problems involving space and form. It also deals with relationship between magnitudes. It enables the man to study various phenomena in space and establish various relations hips between them. It explains that this science is a by-product of our empirical knowledge. From our observations of physical and social environment, we form certain intuitive ideas or notions called postulates and axioms. By a process of reasoning, we move upwards and work out mathematical results at the abstract level. "Mathematics may also be defined as the science of abstract form. The discernment of structure is essential no less to the appreciation of a painting or a symphony than to understand the behaviour of a physical system; no less in economics than in astronomy. Mathematics studies order abstracted from the particular objects and phenomena which exhibit it and in a generalised form."

SCIENCE OF LOGICAL REASONING

Mathematics is also called the science of logical reasoning. In it, we approach everything with a question mark in our mind. As Locke has said, "Mathematics

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is a way to settle in the mind a habit of reasoning.” Here the results are developed through a process of reasoning. There are only a few premises on which we base our reasoning. The conclusions follow naturally from the given facts when logical reasoning is applied to the same. The reasoning in mathematics is of peculiar kind and possesses a number of characteristics such as simplicity, accuracy, certainty of results, originality, similarity to the reasoning of life, and verification. These characteristics have been discussed in detail in the relevant chapters of the book.

MATHEMATICAL LANGUAGE AND SYMBOLISM

Another most important characteristic of mathematics which distinguishes it from many other subjects is its peculiar language and symbolism. Lindsay says, “Mathematics is the language of physical sciences and certainly no more marvellous language was ever created by the mind of man.” Man has the ability to assign symbols for objects and ideas. Mathematical language and symbols cut-short the lengthy statements and help the expression of ideas or things in the exact form. Mathematical language is free from verbosity and helps in to the point, clear and exact expression of facts. For example, instead of saying that the square of the sum of two terms is equal to the sum of the square of the fist term, square of the second term and double the product of the terms, we can simply write (a÷b)2=a-i-b2+2ab in symbolic form.

Mathematical results in their symbolic form help in solving numerous complicated problems. Most of the later progress in mathematics depends heavily on the learner’s ability to employ mathematical language and symbolism. It is reasonable to mention here that most of the results of scientific inventions and discoveries are stated through mathematical language and symbolism.

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Addition, subtraction, multiplication, division and equality are indicated by well known symbols. so that they are in a position to understand mathematical processes and conclusions and mathematical literature. Many of them lose interest in the subject because of their inability to understand mathematical language and symbolism. They cram the statements and processes and try to solve problems mechanically. Rather they should be enabled to understand and appreciate precision, brevity, logic, sharpness and beauty of mathematical language.

PURE AND APPLIED MATHEMATICS

PURE MATHEMATICS

Pure mathematics involves systematic and deductive reasoning. It treats only theories and principles without regard to their application to concrete things. It is developed on an abstract, self-contained basis without any regard to any possible kind of practical applications that may follow. It consists of all those assertions as that if such and such proposition is true of anything, such and such another proposition is true of that thing.

APPLIED MATHEMATICS

Applied mathematics is the application of pure mathematics in the service of a given purpose. It has some direct or practical application to objects and happenings in the material world. It plays a great role in the development of various subjects. Every discovery in science owes much to applied mathematics. Principles of applied mathematics have been useful in the investigation of such phenomenon as heat. Sound , light, optics, navigation and astronomy. Applied mathematics is a part of mathematics definitely related to or suggested by some tangible situations, though not always intended for practical use. It is the connecting link between pure mathematics on one side,

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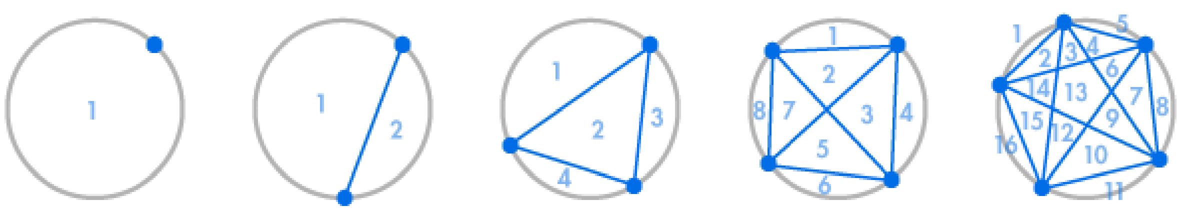
physical, biologic al, social sciences and technology on the other. It acts and reacts not only on science technology but also on pure mathematics e.g., space dynamics, ballistics, fluid dynamics, and elasticity theory, theory of relativity, mathematical biology, and mathematical economics.

BUILDING BLOCKS OF MATHEMATICS- AXIOMS, PROPOSITIONS, POSTULATES, QUANTIFIERS ETC.;

AXIOMS

Introduction

Imagine that we place several points on the circumference of a circle and connect every point with each other. This divides the circle into many different regions, and we can count the number of regions in each case. The diagrams below show how many regions there are for several different numbers of points on the circumference. We have to make sure that only two lines meet at every intersection inside the circle, not three or more.

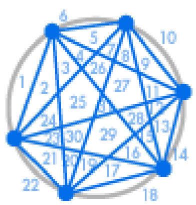


1 region 2 regions 4 regions 8 regions 16 regions

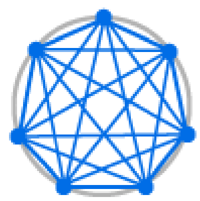
We can immediately see a pattern: the number of regions is always twice the previous one, so that we get the sequence 1, 2, 4, 8, 16, … This means that with

6 points on the circumference there would be 32 regions, and with 7 points there would be 64 regions.

We might decide that we are happy with this result. The number of regions is always twice the previous one – after all this worked for the first five cases. Or we might decide that we should check a few more, just to be safe:



31 regions, not 32



57 regions, not 64

Unfortunately something went wrong: 31 might look like a counting mistake, but 57 is much less than 64. The sequence continues 99, 163, 256, …, very different from what we would get when doubling the previous number.

This example illustrates why, in mathematics, you can't just say that an observation is *always true* just because it works in a few cases you have tested. Instead you have to come up with a rigorous logical argument that leads from results you already know, to something new which you want to show to be true. Such an argument is called a **proof**.

Proofs are what make mathematics different from all other sciences, because once we have proven something we are*absolutely certain* that it is and will *always* be true. It is not just a theory that fits our observations and may be replaced by a better theory in the future.

In the above example, we could count the number of intersections in the inside of the circle. Thinking carefully about the relationship between the number of intersections, lines and regions will eventually lead us to a different equation for the number of regions when there are $x = 3$ points on the circle:

Number of regions $= x^4 - 6x^3 + 23x^2 - 18x + 2424 = 4$.

This equation works in all the cases above. We could now try to prove it for *every* value of x using "induction", a technique explained below.

Traditionally, the end of a proof is indicated using a ■ or □, or by writing *QED* or *"quod erat demonstrandum"*, which is Latin for *"what had to be shown"*.

A result or observation that *we think* is true is called a **Hypothesis** or **Conjecture**. Once we have proven it, we call it a**Theorem**. Once we have proven a theorem, we can use it to prove other, more complicated results – thus building up a growing network of mathematical theorems.

Axioms

One interesting question is where to start from. How do you prove the first theorem, if you don't know anything yet? Unfortunately you can't prove something using nothing. You need at least a few building blocks to start with, and these are called**Axioms**.

Mathematicians assume that axioms are true without being able to prove them. However this is not as problematic as it may seem, because axioms are either*definitions* or clearly obvious, and there are only very few axioms. For example, an axiom could be that $a + b = b + a$ for any two numbers a and b.

Axioms are important to get right, because all of mathematics rests on them. If there are too few axioms, you can prove very little and mathematics would not be very interesting. If there are too many axioms, you can prove almost anything, and mathematics would also not be interesting. You also can't have axioms contradicting each other.

Mathematics is not about choosing the right set of axioms, but about developing a framework from these starting points. If you start with different axioms, you will get a *different kind of mathematics*, but the logical arguments will be the same. Every area of mathematics has its own set of basic axioms.

When mathematicians have proven a theorem, they publish it for other mathematicians to check. Sometimes they find a mistake in the logical argument, and sometimes a mistake is not found until many years later. However, in principle, it is always possible to break a proof down into the basic axioms.

Real Number Axioms

As much as possible, in mathematics we base each field of study on axioms. Axioms are rules that give the fundamental properties and relationships between objects in our study. We do not prove axioms! We take them as mathematical facts and we deduce theorems from them. As long as the axioms don't contradict one another, we can make up any axioms we want to give structure to what we are studying. How and why we choose certain axioms is a more philosophical question than a mathematical one. For the real number system, the axioms are chosen so that the standard properties of numbers (that you are familiar with from grade school) hold.

Field Axioms

A field is a set, S, with operations '+' and '·' such that A0: $x + y \in S$ M0: $x \cdot y \in S$ Closure A1: $(x + y) + z = x + (y + z)$ M1: $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ Associativity A2: $x + y = y + x$ M2: $x \cdot y = y \cdot x$ Commutativity A3: $x + 0 = x$ M3: $x \cdot 1 = x$ Identity A4: Given $x \in S$, there is $(-x) \in S$ M4: For x 6= 0, there is $1/x \in S$ Inverse such that $x + (-x) = 0$ such that $x \cdot (1/x) = 1$ DL: $x \cdot (y + z) = x \cdot y + x \cdot z$ Distribution Notes on the Field Axioms: 1. These properties are satisfied by Q and R (and Zp which we will study at the end of the term). 2. By closure, you can add or multiply any two elements of S as you wish and get another element of S. 3. An 'operation' must always output the same element when given the same inputs (this is called being 'well-defined'). That is, if $x = y$ and z is any element in S, then $x + z = y + z$ and $x \cdot z = y \cdot z$.

Order Axioms

A field, F, is ordered if it contains a positive set P such that: P1: $x, y \in P$ implies $x + y \in P$, P2: $x, y \in P$ implies $x \cdot y \in P$, and P3: $x \in F$ implies $x = 0$, $x \in P$, or $-x \in P$. Notes on Order Axioms: 1. These properties are satisfied by Q and R. Basically, this says that all numbers are positive, negative, or zero and that the sum or product of positive numbers is positive. 2. We define $x < y$ to mean that $y - x$ is positive (i.e. $y - x \in P$). And $x \leq y$ means $y - x \in P$ or $y - x = 0$.

Completeness Axiom

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An ordered field F is complete if every nonempty subset of F that has an upper bound in F has a least upper bound in F. The completeness axiom is satisfied by R and not by Q. It basically says that there are ‘no gaps’ in the set. The set R can be classified as a complete ordered field.

Math Logic

1 Introduction Math is like a foreign language. It is used to describe the world around us. In order to understand math, like any other language you must learn the vocabulary and how to express ideas with that vocabulary. Math logic is the structure that allows us to describe concepts in terms of math. We will start with very basic ideas and build on them. Starting in this way, we will be able to build an understanding of math in manageable steps. With this foundation even more complicated problems won’t be so difficult.

PROPOSITIONS

A proposition is a declaration that can be either true or false, but not both. For example, “Today is Friday” is a proposition. This statement can be true or false, but not both. It is common to define a shorthand notation for propositions: Let P be the proposition “Today is Friday.” If the statement is true, then P has truth value true. If it is false, then P has truth value false. It is also common notation to use a “place filler” in a proposition. For example let P(x) be “x is a an odd number.” Then P(x) is a proposition depending on x. Sample problems: Determine which of the following are propositions. 1. x = 5. 2. The grey dog. 3. The dog is brown. 4. The real numbers. Of course the propositions that we have been using here aren’t sufficiently complicated to develop meaningful mathematical ideas. So we must describe interactions with propositions.

Naturally, we define an operation called negation. This idea is exactly what you would expect it to be. For example, let P denote “Today is Friday”. Then the negation of P, written $-P$, is “Today is not Friday.” Notice that P and $-P$ cannot both have the same truth value. This is called the law of non contradiction. Sample problems: Find the negation of each of the previous sample problems determined to be propositions. Next we define two operations on propositions. Again these operations are intuitive based on every day vernacular. The operations are “and”, denoted “$\wedge$”, and “or”, denoted “$\vee$.” In order for the proposition $P \wedge Q$ to be true, both P and Q must be true. In order for $P \vee Q$ to be true either P must be true or Q must be true, not necessarily both.

For example, let P be “$x < 4$” and Q be “$x > 2$.” Then $P \wedge Q$ is the proposition “$x < 4$ and $x > 2$.” And $P \vee Q$ is the proposition “$x < 4$ or $x > 2$.” Sample problems: Write the following in words based on the propositions: Let P be “I have a sister,” Q be “I have a brother,” and N be “I have a cousin.” 1. $P \wedge Q \wedge N$ 2. $P \wedge -Q$ 3. $P \vee Q$ 4. $(P \vee Q) \wedge N$

3 Implication

Implication or logical implication is another relationship between two propostions. Implication is basically the idea of an if, then statemnt. For example, let P be “Today is Saturday” and Q be “It is the weekend.” Then the implication, “P implies Q”, written $P \Rightarrow Q$, means “If it is Saturday, then it is the weekend.” Then we can determine that $P \Rightarrow Q$ is true. Note that $Q \Rightarrow P$, called the converse, is false. Sample problems: Write the following in words based on the propositions: Let P be “$x = 1$” and Q be “x 2 = 1.” Then determine which are true. 1. $P \Rightarrow Q$ 2. $Q \Rightarrow P$ It is also possible for $P \Rightarrow Q$ and $Q \Rightarrow P$ to both be true. So when $P \Rightarrow Q$ and $Q \Rightarrow P$, we write $P \Leftrightarrow Q$ and say “P if and

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only if Q." When this condition is true, we also say that P and Q are equivalent. Implication can be written in terms of the previously defined operations. The symbolic definition of implication is: $P \Rightarrow Q = -P \vee Q$. Then it can be seen that $P \Rightarrow Q = -P \vee Q = Q \vee -P = -Q \Rightarrow -P$. So $P \Rightarrow Q$ and $-Q \Rightarrow -P$ are equivalent or $(P \Rightarrow Q) \Leftrightarrow (-Q \Rightarrow -P)$. This equivalent statement is called the contrapositive. Sample Problems: Write the contrapositive in words of the implications in the previous sample problems.

QUANTIFIERS

The previous sections have made it possible to write a lot of different propositions, but we are still not able to write all of the statements that we will want. Quantifiers allow us to specify the scope of the propositions that we can write. There are two quantifiers that we will be concerned with. The first is called the universal quantifier, written ∀. This quantifiers is used to define the scope of a proposition. For example, the proposition "All people are good" can be written symbolically using the universal quantifier. Let P(x) be the statement "x is good." Then the previous proposition can be written as "∀ x is a person, P(x)." The other quantifier that we introduce is the existential quantifier, written ∃. This quantifier is also used to define the scope of a proposition. For example the proposition "There exist good people" can be written symbolically "∃ a person x such that P(x)." It is often useful to use these two together to write propositions. For example the proposition "For any real number x, there is a real number that is greater than x" can be written symbolically as: "∀ real number x, ∃ a real number y such that $y > x$." Sample problems: Define propositions and use quantifiers to write the following symbolically.

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1. The square of every real number is non-negative.

2. The square root of any positive number is a real number.

3. Every real number has an additive inverse.

4. Every non-constant linear function has a zero

NATURE OF MATHEMATICS- TRUTH,

Truth in mathematics We turn finally to the discussion of how closely mathematical reasoning captures the essential nature of truth. In order to avoid repeated circumlocutions, I shall speak of mathematical truth. It is first necessary to examine the way that mathematics is presented. Refer to example 1. People do make mistakes. Is this significant? I maintain that it is. Anyone who has read the mathematical literature to any extent knows that it is riddled with errors. It is reasonable to expect that a random mathematical paper, which is, say, ten pages long, will contain at least one error in reasoning. There is even an aphorism to describe the situation: The way to tell a good mathematician from a mediocre mathematician is that his results are right, even though his proofs are wrong. The existence of errors in reasoning is so rampant that mathematicians have developed a peculiarly diffident manner of speaking to each other, which is illustrated in the following mythical conversation: A. “Is thus and so the case?” B. “I think so. I think I have a proof.” Errors in reasoning occur with the best of us. Lebesgue in 1905 published a “theorem” which is so “obviously” wrong that no competent graduate student today would believe it. [The projection of a plane Borel set on the line is a Borel set.] It was not until 1916 that Souslin showed that the result is incorrect. Let me mention what may be the extreme case of an act of faith in the ability of humans to carry out a chain of reasoning without error. There is a theorem due to Thompson and Feit.

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[All finite groups of odd order are solvable.] It is believed and has been very influential. It has inspired a large amount of activity in group theory. Its proof occupies a whole paper. The paper is 254 pages long. There is a footnote on the first page which says: "The authors are grateful to Professor E. C. Dade,

THE NATURE OF TRUTH whose careful study of a portion of this paper has disclosed several blunders." It is likely that no one has made a careful study of the whole paper (including the authors, separately). Every mathematician knows of the existence of this phenomenon, even if it is only a personal feeling. No one escapes the nightmare that everything he has done is wrong. But by a universal schizophrenia, mathematicians feel that this has nothing to do with mathematical truth. The feeling is that if results or arguments are wrong, then since they are in the public domain, this will be pointed out and acknowledged. The argument is intolerable. First of all, for most results it is hard to specify an impartial audience who will greet the result with anything but apathy. However, even more important is that the status of a mathematical truth is then that no one has yet shown this to be wrong. You can do that well without proofs. Even the results of physics, which are customarily considered less secure than mathematical results, are presented with better evidence. There is a better argument in favor of mathematical truth, and I now turn to it. The argument is this. It is unfair to count human frailty against the "essential" nature of mathematical truth. Mathematics is essentially axiomatic. One starts from a fixed set of axioms and by making use of accepted rules of reasoning arrives at conclusions. The correct form for a mathematical result is that if the axioms hold, then the conclusions hold. These results are inescapable. The fact that mathematics is usually not presented in this fashion is perhaps unfortunate; and

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the fact that the rules of reasoning are not all applied correctly is certainly unfortunate; but nevertheless this does not alter the “essential” nature of things. The first point in this argument which I will examine is the phrase, “accepted rules of reasoning.” What are they? The nature of affairs concerning “accepted rules of reasoning” is, I feel, more astonishing than the fact that the mathematical literature is riddled with errors. You will have great difficulty in finding a mathematician who will tell you what the “accepted rules of reasoning” are. It is unusual in the training of a mathematician to have anyone discuss them. They are learned by observing examples of their use while other material is being presented. Some of them are rather strange. It is not unusual to find a good student who boggles at one or more of them. Since the student is very unlikely to find anyone who understands his qualms, it is usual for him to be left with the uncomfortable feeling that something is wrong either with him or with the world. (But, of course, nothing is wrong with mathematical truth.) There is a seeming inconsistency in what I have been saying. If the “accepted rules of reasoning” are never stated, how can I talk about them as if there were a fixed set of rules? The answer is that they have been stated. However, the statement is made by logicians. Logic is not a popular subject with mathematicians, and it is unusual to find a mathematician who is conversant with the results obtained. Let me pause to state what it is that logicians have done. Logicians have created a model of mathematical reasoning. It is called a formal language. The language is completely specified. A proof is a finite sequence of sentences in the language, which has the property that each sentence is either one of a prescribed set of axioms or follows from preceding sentences in a way which is susceptible to checking by a mechanical procedure. It is this development which is at the basis of current work in theorem proving

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(or checking) by computer. The work in this area has been substantial, and there are reasons to believe that formal languages have captured the essence of mathematical reasoning. Does the existence of formal languages put mathematical truth on unshakable ground? I don't think so. First, most mathematicians are unaware of the existence of formal languages. Even if they were aware of it, they certainly would not carry out their arguments in a formal language. Second, this abstract presentation of the "accepted rules of reasoning" is rather complicated.

LOGIC,

MATHEMATICS AS A BRANCH OF LOGIC

As was pointed out earlier, all the theorems of arithmetic, algebra, and analysis can be deduced from the Peano postulates and the definitions of those mathematical terms which are not primitives in Peano's system. This deduction requires only the principles of logic plus, in certain cases, the axiom of choice. By combining this result with what has just been said about the Peano system, the following conclusion is obtained, which is also known as *the thesis of logicism concerning the nature of mathematics*:

Mathematics is a branch of logic. It can be derived from logic in the following sense:

a. All the concepts ot mathematics, i.e., of arithmetic, algebra, and analysis, can be defined in terms of four concepts of pure logic.

b. All the theorems of mathematics can be deduced from those definitions by means of the principles of logic (including the axioms of infinity and choice).[10]

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In this sense it can be said that the propositions of the system of mathematics as here delimited are true by virtue of the definitions of the mathematical concepts involved, or that they make explicit certain characteristics with which we have endowed our mathematical concepts by definition. The propositions of mathematics have, therefore, the same unquestionable certainty which is typical of such propositions as "All bachelors are unmarried," but they also share the complete lack of empirical content which is associated with that certainty: The propositions of mathematics are devoid of all factual content; they convey no information whatever on any empirical subject matter.

REASONING,

Reasoning is fundamental to knowing and doing mathematics. We wonder how you would define the term? Some would call it systematic thinking. Reasoning enables children to make use of all their other mathematical skills and so reasoning could be thought of as the 'glue' which helps mathematics makes sense.

The second aim of the new mathematics national curriculum in England (DfE, 2013) is that all pupils will:

reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.

Deductive and Inductive Reasoning

You have probably heard the words **inductive** and **deductive** reasoning many times before. Chances are you have even mixed the two up, or think they mean the same thing. But, they are not the same thing. In fact, they are actually opposites!

You can think of inductive and deductive reasoning as a path from something you know, to something you don't know. Each makes use of general knowledge and knowledge of a specific scenario - think of it like the difference between a large lake and an individual fish living in the lake. The beginning point and ending points are switched on each.

This lesson will introduce each type of reasoning through definitions, general examples and mathematical examples. Join me as we learn to reason.

Inductive Reasoning - Definition

Inductive reasoning starts with a specific scenario and makes conclusions about a general population. For our lake example, if you found a trout fish in a lake, you would assume that it is not the only fish in that lake. You may further conclude that all the fish in the lake are trout. You have induced from specific scenario information about the general population of the lake.

An interesting point with induction is that it allows for the conclusion to be false. It is simply a process of logical reasoning from a specific observation to a general theory of a population. The conclusion that all the fish in the lake are trout is very likely to be wrong; however, the process of induction was pure and logical. You could call it a valid guess.

PROBLEM SOLVING;

Problem solving is a major focus of the mathematics curriculum; engaging in mathematics *is* problem solving. Problem solving is what one does when a solution is not immediate. Students should build mathematical knowledge through problem solving, develop abilities in formulating and representing problems in various ways, apply a wide variety of problem-solving strategies, and monitor their mathematical thinking in solving problems. Problems become the context in which students develop mathematical under-standings, apply skills, and generalize learning. Students frequently solve problems in cooperative groups and even create their own problems.

Students should learn to **reason and construct proofs** as essential and powerful aspects of understanding and using mathematics. These processes involve making and investigating conjectures, developing and evaluating arguments, and applying various types of reasoning and methods of proof. Reasoning skills are critical for science, social studies, social skills, literature, and most other areas of study.

SALE

Converse, Inverse, Contrapositive

Given an if-then statement "if *p*, then *q*", we can create three related statements:

A conditional statement consists of two parts, a hypothesis in the "if" clause and a conclusion in the "then" clause. For instance, "If it rains, then they cancel school."

"It rains" is the hypothesis.

"They cancel school" is the conclusion.

To form the converse of the conditional statement, interchange the hypothesis and the conclusion.

The converse of *"If it rains, then they cancel school"* is *"If they cancel school, then it rains."*

To form the inverse of the conditional statement, take the negation of both the hypothesis and the conclusion.

The inverse of *"If it rains, then they cancel school"* is *"If it does not rain, then they do not cancel school."*

To form the contrapositive of the conditional statement, interchange the hypothesis and the conclusion of the inverse statement.

The contrapositive of *"If it rains, then they cancel school"* is *"If they do not cancel school, then it does not rain."*

Statement	If *p*, then *q*.
Converse	If *q*, then *p*.
Inverse	If not *p*, then not *q*.
Contrapositive	If not *q*, then not *p*.

If the statement is true, then the contrapositive is also logically true. If the converse is true, then the inverse is also logically true.

Example 1:

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Statement	If two angles are congruent, then they have the same measure.
Converse	If two angles have the same measure, then they are congruent.
Inverse	If two angles are not congruent, then they do not have the same measure.
Contrapositive	If two angles do not have the same measure, then they are not congruent.

In the above example, since the hypothesis and conclusion are equivalent, all four statements are true. But this will not always be the case!

Example 2:

Statement	If a quadrilateral is a rectangle, then it has two pairs of parallel sides.
Converse	If a quadrilateral has two pairs of parallel sides, then it is a rectangle. **(FALSE!)**
Inverse	If a quadrilateral is not a rectangle, then it does not have two pairs of parallel sides. **(FALSE!)**
Contrapositive	If a quadrilateral does not have two pairs of parallel sides, then it is not a

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	rectangle.

PROOFS AND TYPES OF PROOFS,

Types of Proofs.

Suppose we wish to prove an implication $p \to q$. Here are some strategies we have available to try.

• Trivial Proof: If we know q is true then $p \to q$ is true regardless of the truth value of p.

• Vacuous Proof: If p is a conjunction of other hypotheses and we know one or more of these hypotheses is false, then p is false and so $p \to q$ is vacuously true regardless of the truth value of q.

• Direct Proof: Assume p, and then use the rules of inference, axioms, defi- nitions, and logical equivalences to prove q.

• Indirect Proof or Proof by Contradiction: Assume p and $\neg q$ and derive a contradiction $r \wedge \neg r$.

•

Proof by Contrapositive: (Special case of Proof by Contradiction.) Give a direct proof of $\neg q \to \neg p$. Assume $\neg q$ and then use the rules of inference, axioms, definitions, and logical equivalences to prove $\neg p$.(Can be thought of as a proof by contradiction in which you assume p and $\neg q$ and arrive at the contradiction $p \wedge \neg p$.)

• Proof by Cases: If the hypothesis p can be separated into cases $p1 \vee p2 \vee \cdots \vee pk$, prove each of the propositions, $p1 \to q$, $p2 \to q$, . . . , $pk \to q$, separately. (You may use different methods of proof for different cases.) Discussion We are now getting to the heart of this course: methods you can use to write proofs. Let's investigate the strategies given above in some detail. 2.2. Trivial Proof/Vacuous Proof.

Example 2.2.1. Prove the statement: If there are 100 students enrolled in this course this semester, then 6 2 = 36.

Proof. The assertion is trivially true, since the conclusion is true, independent of the hypothesis (which, may or may not be true depending on the enrollment).

Example 2.2.2. Prove the statement. If 6 is a prime number, then 6 2 = 30.

Proof. The hypothesis is false, therefore the statement is vacuously true (even though the conclusion is also false). Discussion The first two methods of proof, the "Trivial Proof" and the "Vacuous Proof" are certainly the easiest when they work. Notice that the form of the "Trivial Proof", $q \to (p \to q)$, is, in fact, a tautology. This follows from disjunction introduction, since $p \to q$ is equivalent to $\neg p \lor q$. Likewise, the "Vacuous Proof" is based on the tautology $\neg p \to (p \to q)$.

Exercise 2.2.1. Fill in the reasons for the following proof of the tautology $\neg p \to (p \to q)$. $[\neg p \to (p \to q)] \Leftrightarrow [p \lor (\neg p \lor q)] \Leftrightarrow [(p \lor \neg p) \lor q] \Leftrightarrow T \lor q \Leftrightarrow T$ Exercise 2.2.2. Let $A = \{1, 2, 3\}$ and $R = \{(2, 3),(2, 1)\}(\subseteq A \times A)$. Prove: if $a, b, c \in A$ are such that $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$. Since it is a rare occasion when we are able to get by with one of these two methods of proof, we turn to some we are more likely to need. In most of the following examples the underlying "theorem" may be a fact that is well known to you. The purpose in presenting them, however, is not to surprise you with new mathematical facts, but to get you thinking about the correct way to set up and carry out a mathematical argument, and you should read them carefully with this in mind.

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Direct Proof.

Example 2.3.1. Prove the statement: For all integers m and n, if m and n are odd integers, then m + n is an even integer. Proof. Assume m and n are arbitrary odd integers. Then m and n can be written in the form m = 2a + 1 and n = 2b + 1,

where a and b are also integers. Then m + n = (2a + 1) + (2b + 1) (substitution) = 2a + 2b + 2 (associative and commutative laws of addition) = 2(a + b + 1) (distributive law) Since m+n is twice another integer, namely, a+b+1, m+n is an even integer.

Proof by Contrapositive.

Example 2.4.1. Prove the statement: For all integers m and n, if the product of m and n is even, then m is even or n is even. We prove the contrapositive of the statement: If m and n are both odd integers, then mn is odd. Proof. Suppose that m and n are arbitrary odd integers. Then m = 2a + 1 and n = 2b + 1, where a and b are integers. Then

mn = (2a + 1)(2b + 1) (substitution) = 4ab + 2a + 2b + 1 (associative, commutative, and distributive laws) = 2(2ab + a + b) + 1 (distributive law) Since mn is twice an integer (namely, 2ab + a + b) plus 1, mn is odd.

Proof by Contradiction.

Example 2.5.1. Prove the statement is true: Let x and y be real numbers. If 5x + 25y = 1723, then x or y is not an integer. Proof. Assume x and y are real numbers such that 5x+25y = 1723, and assume that both x and y are integers. By the distributive law, 5(x + 5y) = 1723. Since x and y are integers, this implies 1723 is divisible by 5. The integer 1723, however, is clearly not divisible by 5. This contradiction establishes the result

Proof by Cases.

Example 2.6.1. If x is a real number such that x 2 − 1 x + 2 > 0, then either x > 1 or −2 < x < −1. Proof. Assume x is a real number for which the inequality x 2 − 1 x + 2 > 0 holds. Factor the numerator of the fraction to get the inequality (x + 1)(x − 1) x + 2 > 0. For this combination of x + 1, x − 1, and x + 2 to be positive, either all are positive or two are negative and the other is positive. This gives four cases to consider:

Case 1. $x + 1 > 0$, $x - 1 > 0$, and $x + 2 > 0$. In this case $x > -1$, $x > 1$, and $x > -2$, which implies $x > 1$.

Case 2. $x + 1 > 0$, $x - 1 < 0$, and $x + 2 < 0$. In this case $x > -1$, $x < 1$, and $x < -2$, and there is no x satisfying all three inequalities simultaneously.

Case 3. $x + 1 < 0$, $x - 1 > 0$, and $x + 2 < 0$. In this case $x < -1$, $x > 1$, and $x < -2$, and there is no x satisfying all three inequalities simultaneously.

Case 4. $x + 1 < 0$, $x - 1 < 0$, and $x + 2 > 0$. In this case $x < -1$, $x < 1$, and $x > -2$, which implies that $-2 < x < -1$. Thus, either $x > 1$ (Case 1) or $-2 < x < -1$ (Case 4).

AESTHETICS BY BIRKHOFF.

HISTORY OF MATHEMATICS WITH SPECIAL EMPHASIS ON TEACHING OF MATHEMATICS, CONTRIBUTION OF MATHEMATICIANS- ARYABHATTA, RAMANJUNAN, PYTHAGORAS & EUCLID.

Indian Mathematicians

RAMANUJAN

He was born on 22^{na} of December 1887 in a small village of Tanjore district, Madras. He failed in English in Intermediate, so his formal studies were stopped but his self-study of mathematics continued.

He sent a set of 120 theorems to Professor Hardy of Cambridge. As a result he invited Ramanujan to England.

Ramanujan showed that any big number can be written as sum of not more than four prime numbers.

He showed that how to divide the number into two or more squares or cubes.

when Mr Litlewood came to see Ramanujan in taxi number 1729, Ramanujan said that 1729 is the smallest number which can be written in the form of sum of cubes of two numbers in two ways, i.e. $1729 = 9^3 + 10^3 = 1^3 + 12^3$ since then the number 1729 is called Ramanujan's number.

In the third century B.C, Archimedes noted that the ratio of circumference of a circle to its diameter is constant. The ratio is now called 'pi (Π)' (the 16th letter in the Greek alphabet series)

The largest numbers the Greeks and the Romans used were 106 whereas Hindus used numbers as big as 10^{53} with specific names as early as 5000 B.C. during the Vedic period.

ARYABHATA

Aryabhatta was born in 476A.D in Kusumpur, India.

He was the first person to say that Earth is spherical and it revolves around the sun.

He gave the formula $(a + b)^2 = a^2 + b^2 + 2ab$

He taught the method of solving the following problems:

BRAHMAGUPTA

Brahma Gupta was born in 598A.D in Pakistan.

He gave four methods of multiplication.

He gave the following formula, used in G.P series

$a + ar + ar^2 + ar^3 + \ldots\ldots\ldots.. + ar^{n-1} = (ar^{n-1}) \div (r - 1)$

He gave the following formulae :

Area of a cyclic quadrilateral with side a, b, c, d= $\sqrt{}(s -a)(s- b)(s -c)(s- d)$ where $2s = a + b + c + d$ Length of its diagonals =

SHAKUNTALA DEVI

She was born in 1939

In 1980, she gave the product of two, thirteen digit numbers within 28 seconds, many countries have invited her to demonstrate her extraordinary talent.

In Dallas she competed with a computer to see who give the cube root of 188138517 faster, she won. At university of USA she was asked to give the 23^{rd} root of 916748676920039158098660927585380162483106680144308622407126516427 934657040867096593279205767480806790022783016354924852380335745316935111903596577547340075681868830562082101612913284556489578015880677 1.

UNIT II: AIMS, OBJECTIVES AND APPROACHES TO TEACHING SCHOOL MATHEMATICS

AIMS AND OBJECTIVES OF MATHEMATICS:

Education is imparted for achieving certain ends and goals. Various subjects of the school curriculum are different means to achieve these goals. The term aims of teaching mathematics stands for the goals, targets or broader purposes that may be full filled by the teaching of mathematics in the general scheme of education. Aims are like ideals. Their attainment needs a long-term planning. Their realisation is not an easy task. Therefore, they are divided into some definite, functional and workable units named as objectives. The objectives are those short-term, immediate goals or purposes that may be achieved within the specified classroom situation. They help in bringing about behavioural changes in the learners for the ultimate realisation of the aims of teaching mathematics. The aims are broken into specified objectives to provide definite learning experiences for bringing about desirable behavioural changes.

1. Utilitarian aim

Mathematics will be taught primarily for its practical values and aims. The student will be given mathematical knowledge and skills needed in his day-today life and enabled to make use of that knowledge and skill. This aim makes the study of mathematics functional and purposeful and establishes relation between the subject and practical life.

2. Disciplinary aim

The subject has also to be taught for its disciplinary and intellectual values. It has to aim at providing training to the mind of the learner and developing intellectual habits in him.

3. Cultural aim

T.iis aim helps the learner to understand the contribution of mathematics in the development of civilisation and culture. It has enabled him to understand the role of mathematics in fine arts and in beautifying human life.

4. Adjustment aim

It is to help the learner to develop a healthy, purposeful, productive, exploratory and controlling adjustment with environment.

S. Social aim

It is to help the learner to imbibe essential social virtues.

6. Moral aim

It enables the learners to imbibe the attributes of morality.

7. Aesthetic aim

It is to develop their aesthetic sensibilities, meet their varying interests and help them in the proper utilisation of their leisure time.

8. International aim

To develop in them international outlook and understanding.

9. Vocational aim

It is to prepare them for technical and other vocations where mathematics is applied.

10. Inter-disciplinary aim

To give them insight into the application of mathematics in other subjects.

11. Self-education aim

It is to help them to become independent in learning.

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12. Educational preparation aim

h is to prepare them for higher education in sciences, engineering, technology, etc.

13. Development of powers aim

It pertains to the development of powers of thinking, reasoning, concentration, expression, discovery. etc.

14. Harmonious development aim

Ultimately the overall aim of teaching all the subjects including mathematics is to ensure alround and harmoneous development of the personality of the child.

OBJECTIVES AT THE ENTIRE SCHOOL STAGE

The objectives of teaching mathematics at the entire school stage or secondary stage may be classified as under:

a) Knowledge and understanding objectives.

b) Skill objectives.

c) Application objectives.

d) Attitude objectives.

e) Appreciation and interest objectives.

To make them unambiguous and attainable, these objectives are further e pressed in behavioural terms. What the student is expected to achieve is clearly known to the teacher in the form of desirable behavioural changes.. l)different objectives along with the relevant behavioural changes are given Below:

A. KNOWLEDGE AND UNDERSTANDING OBJECTIVES:

The student acquires knowledge and understanding of:

I. *Language* of Mathematics**.** i.e. , the language of its technical terms, symbols, statements, formulae, definitions, logic, etc.

2. *Various Concepts.* i.e. , concept of number, concept of direction, conc ept of measurement.

3. *Mathematical Ideas*, like facts, principles, processes and relationships.

4. The development of the subject over the centuries and contributions of mathematicians.

5. Inter-relationship between different branches and topics of mathematic, etc.

6. The nature of the subject of mathematics.

B. SKILL OBJECTIVES

The subject helps the student to develop the following skills:

1. He acquires and develops skill in the use and understanding of mathematical Language.

2. He acquires and develops speed, neatness, accuracy, and brevity and precision in mathematical calculations.

3. He learns and develops technique **of** problem-solving.

4. He develops the ability to estimate, check and verify results.

5. He develops the ability to perform calculations orally and mentally.

6. He develops ability to think correctly to draw conclusions, generalisations and inferences.

7. He develops skills to use mathematical tools, and apparatuses.

8. He develops essential skill in drawing geometrical figures.

9. He develops skill in drawing, reading, interpreting graphs and statistical tables.

10. He develops skill in measuring, weighing and surveying.

II. He develops skill in the use of mathematical tables and ready references.

C. APPLICATION Objectives:

The subject helps the student to apply the above mentioned knowledge am' sills in the following way:

I. He is able to solve mathematical problems independently.

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2. He makes use of mathematical concepts and processes in everyday life.

3. He develops ability to analyse, to draw inferences, and to generalise from the collected data and evidence.

4. He can think and express precisely, exactly, and systematically by making proper use of mathematical language.

5. He develops the ability to use mathematical knowledge in the learning of other subjects especially sciences.

6. He develops the students' ability to apply mathematics in his future vocational life.

D) ATTITUDE OBJECTIVES :

The subject helps to develop the following attitudes:

1. the student learns to analyse the problems.

2. Develops the habit of systematic thinking and objective reasoning.

3. He develops heuristic attitude and tries to discover solutions and proof **with** his own independent efforts.

4. He tries to collect enough evidence for drawing inferences and generalisations.

5. He recognises the adequacy or inadequacy of given data in relation to any problem.

6. He verifies his results.

7. He understands and appreciates logical, critical and independent thinking in others.

8. He expresses his opinions precisely, accurately, logically and object ively without any biases and prejudices.

9. He develops personal qualities. e.g. , regularity, honesty, objectivity, nearness and truthfulness.

1O. He develops self-confidence for solving mathematical problems.

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II. He develops mathematical perspective and outlook for observing the realm of nature and society.

12. He shows originality and creativity.

E. APPRECIATION AND INTEREST OBJECTIVES

The student is helped in the acquisition of appreciations and interests in th: following way:

I. He appreciates the role of mathematics everyday life.

2. He appreciates the role of mathematics in understanding his environment.

3. He appreciates mathematics as the science of all sciences and art of all arts.

4. He appreciate the contribution made by mathematics in the development of civilisation and culture.

5. He appreciate the aesthetic nature of mathematics by observing symmetry. Similarity , order and arrangement in mathematical facts, principles and processes.

6. He appreciates the contribution of mathematics in the development of other branches of knowledge.

7. He appreciates the recreational values of the subject and learns to utilise it in his leisure time.

8. He appreciates the vocational value of mathematics.

9. He appreciates the role of mathematical language, graphs and tables in giving precision and accuracy to his expression.

10. He appreciates the power of computation developed through the subject.

11. Appreciates the role of mathematics in developing his knowledge.

12. He appreciates mathematical problems, their intricacies and difficulties.

13. He develops interest in the learning of the subject**.**

14. He feels entertained **by** mathematical recreations.

15. He takes an active interest in the activities of mathematics club.

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16. He takes an active interest in active library reading, mathematical projects, and doing practical work in mathematics laboratory.

OBJECTIVES OF TEACHING Mathematics AT DIFFERENT STAGES

Th**e** aims and objectives of teaching mathematics at the entire school stage have been discussed above. However, the aims and objective**s** in respect of the two stages—elementary and secondary are being discussed below:

OBJECTIVES AT THE SECONDARY STAGE

These objectives are more or less the same as those enlisted for the entire school stage. Still the discussion can be supplemented as under**:**

A. KNOWLEDGE AND UNDERSTANDING OBJECTIVES

1. He understands the inter-relationship of mathematical facts, formulae**,** principles and processes.

2. He understands the theoretical and abstract aspects of mathematics.

B. Skill OBJECTIVES

I. He develops skill in solving the same problem by various possible methods.

C. APPUCATION Objectives

I. He learns the application of mathematics in his day-to-day, social, vocational, occupational and recreational life.

D. APPRECIATION AND INTEREST OBJECTIVES

I. Develops interest in the learning of mathematics.

2. Appreciates the contribution of mathematicians and gets inspiration from their work.

3. Appreciates the power of computational skills.

4. Appreciates and takes interest in using his knowledge of mathematics in solving problems of daily life.

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5. Appreciates the recreational value of mathematics and learns to utilise his leisure time properly.

FORMULATION OF OBJECTIVES

Objectives have to be formulated in respect of every topic we are going to teach. These objectives provide a definite direction to the teacher for the planning of his work. They help him to determine what to teach, how to teach. How to illustrate and how to test.

It is not practicable to supply readymade objectives for each and every unit or topic of the subject. A teacher has therefore to learn the task of formulation of objectives for himself. While framing these objectives, he will keep in mind the following things:

1The nature of the subject matter, i.e., the unit to be taught.

2. The needs and interests of the students, and

3. The availability of resources.

He will search for the specific objectives of teaching a topic from within the broad outlines supplied by the objectives of teaching mathematics in general.

The objectives have to be properly described and expressed in terms of the expected behavioural changes or the learning outcomes. They should not appear too general and vague. They should indicate clearly what the student is expected to achieve through the learning of a unit. They should also fulfil the specific purposes of learning that particular unit. They should be testable so that the teacher can have proper assessment of the learning outcomes.

Now we illustrate this procedure with the help of a particular learning unit.

The formula $(a - b)^2 = a^2 + b^2 - 2ab$.

A. KNOWLEDGE AND UNDERSTANDING OBJECTIVES

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I. The student recalls the knowledge of algebraic multiplication and squaring.

2. He recognises the meanings of the formula in hand.

3. He understands and describes the relationship between the two sides of **t**he above equation.

4. He understands the relationship between this formula and the formulae learnt earlier.

5. He understands and expresses the formula in the form of a diagram and through various other substitutions.

B. SKILL OBJECTIVES

I. He can prove the formula by multiplication**.**

2. He can prove its substituted versions.

3. He can verify its accuracy by various substitutions.

4. He can draw a diagram to represent the formula**.**

5. He can establish relationship between the two formulae:

(a + b)2= a2+ b2+ 2ab

and

(a — b)2= a2+ b2— 2ab

C. APPLICATION OBJECTIVES

1. He can solve new problems, independently by applying the formula.

2. He can work out the geometrical proof of the formula**.**

3. He can locate the life situations where the formula may be applicable.

4. He can construct on his own some problems based on the formula**.**

D. ATTITUDE OBJECTIVES

I. He proves the formula through systematic steps and objective reasoning**.**

2. He solves relevant problems with confidence**.**

3. He develops curiosity for the use and application of the formula**.**

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4. He demonstrates originality and creativity.

E. APPRECIATION AND INTEREST OBJECTIVES

I. Appreciates the nature of the formula and that of the preceding formula**.**

2. He appreciates the application of the formula in promptly solving the relevant problems.

3. He appreciates the use of this formula in learning other topics and branches of mathematics**.**

4. He appreciates the recreational value of the formula**.**

5. Appreciates various diagrammatic and other versions of the formula.

6. Develops interest for learning more and more about the formula and its applications.

Trigonometry specific Objectives

Students will apply various concepts of right triangle trigonometry.

• Use a calculator. • Evaluate the six trigonometric functions for a given angle • Apply the definitions of angle of elevation and angle of depression in real life applications. • Apply the six trig ratios of an acute angle in a right triangle. • Solve a right triangle using Pythagorean Theorem, Right Triangle Definitions of the Trig Functions, and the Triangle Sum Theorem.

Students will apply the concepts of trigonometry to any angle in the rectangular coordinate plane.

• Convert angle measures between degrees and radians. • Verify trig identities. • Use the fundamental identities of trig. • Find the reference angle for a given angle.

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• Apply the properties of reference angles in appropriate situations. • Measure and draw angles of positive and negative rotation. • Apply the six trig ratios when given a point on a terminal side of an angle. • Calculating arc length, sector area, linear and angular velocity.

Students will construct and analyze the graphs of trig functions and their inverses.

• Use a calculator. • Evaluate the inverse of a trig function. • Graph the sine and cosine functions. • Analyze the graph of a sine or cosine function to determine the amplitude, period, and transformations (translation and reflection). • Analyze the equation of a sine or cosine function to determine the amplitude, period, and transformations (translation and reflection). • Evaluate inverse functions. • The graphs of the tangent, cotangent, secant and cosecant function.

Students will verify trig identities and solve trig equations.

• Solve first degree trig equations over the restricted interval of 00 < angle < 3600 • Verify trig identities. • Use the fundamental identities of trig. • Solving second degree trig equations over the restricted interval of 00 < angle < 3600 • Applying trig identities while solving trig equations

Students will solve and find the area of oblique triangles.

• Calculate the area of an oblique triangle. • Apply the law of sines and the law of cosines when finding the sides and angles of an oblique triangle. • Using the law of sines in an ambiguous case. Chapter 8 Students will work with complex numbers in algebraic and trigonometric form. • Work with complex numbers in algebraic and trig form. • Convert complex numbers between both forms • Perform mathematical operations on complex numbers in either form • Plot a complex number and find its absolute value

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Specific objectives of algebra

The student, given rational, radical, or polynomial expressions, will

a) add, subtract, multiply, divide, and simplify rational algebraic expressions;

b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;

c) write radical expressions as expressions containing rational exponents and vice versa; and d) factor polynomials completely.

The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first n terms, finding the nth term, and evaluating summation formulas.

The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of i, and identify field properties that are valid for the complex numbers. Equations and Inequalities

The student will solve, algebraically and graphically,

a) absolute value equations and inequalities;

b) quadratic equations over the set of complex numbers;

c) equations containing rational algebraic expressions; and

d) equations containing radical expressions. Graphing calculators will be used for solving and for confirming the algebraic solutions.

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The student will solve nonlinear systems of equations, including linearquadratic and quadratic, quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions. Functions

The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

The student will investigate and analyze functions algebraically and graphically. Key concepts include

domain and range, including limited and discontinuous domains and ranges;

b) zeros;

c) x- and y-intercepts;

d) intervals in which a function is increasing or decreasing;

e) asymptotes;

f) end behavior;

g) inverse of a function; and

h) composition of multiple functions. Graphing calculators will be used as a tool to assist in investigation of functions.

The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.

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Specific objectives of geometry

Properties of Angles

- Identify parallel and perpendicular lines.
- Find measures of angles.
- Identify complementary and supplementary angles.

Triangles

- Identify equilateral, isosceles, scalene, acute, right, and obtuse triangles.
- Identify whether triangles are similar, congruent, or neither.
- Identify corresponding sides of congruent and similar triangles.
- Find the missing measurements in a pair of similar triangles.
- Solve application problems involving similar triangles.

The Pythagorean Theorem

- Use the Pythagorean Theorem to find the unknown side of a right triangle.
- Solve application problems involving the Pythagorean Theorem.

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ACTIVITY METHOD

Activity method is a technique adopted by a teacher to emphasize his or her method of teaching through activity in which the students participate rigorously and bring about efficient learning experiences. It is a child-centered approach. It is a method in which the child is actively involved in participating mentally and physically. Learning by doing is the main focus in this method. Learning by doing is imperative in successful learning since it is well proved that more the senses are stimulated, more a person learns and longer he/she retains.

Pine G (1989) mentions that in an activity based teaching, learners willingly with enthusiasm internalize and implement concepts relevant to their needs.

So our understanding on the activity method by now should mean any learning that is carried out with a purpose in a social environment, involving physical and mental action, stimulating for creative action or expression.

Why do we need to use activity based learning method?

The information processing theory in psychology views learners as active investigators of their environment. This theory is grounded in the premise that people innately strive to make sense of the world around them.

In the process of learning, they experience, memorize and understand. Students need to be provided with data and materials necessary to focus their thinking and interaction in the lesson for the process of analyzing the information. Teachers need to be actively involved in directing and guiding the students' analysis of the information.

It requires active problem solving by students in finding patterns in the information through their own investigation and analysis. With continued practice in these processes, students learn not the content of the lesson but also develop many other skills.

- It enhances creative aspect of experience.
- It gives reality for learning.
- Uses all available resources.
- Provides varied experiences to the students to facilitate the acquisition of knowledge, experience, skills and values.
- Builds the student's self-confidence and develops understanding through work in his/her group.
- Gets experiences, develop interest, enriches vocabulary and provides stimulus for reading.
- Develops happy relationship between students and students, teachers and students.

- An activity is said to be the language of the child. A child who lacks in verbal expression can make up through use of ideas in the activity.
- Subjects of all kind can be taught through activity.
- Social relation provides opportunity to mix with others.

Kinds of activities:

The activities used in this strategy can be generalized under three main categories:

- *Exploratory* - gathering knowledge, concept and skill.
- *Constructive* - getting experience through creative works.
- *Expressional* – presentations.

The Activities you could focus on:-

Experiencing:

- watching, observing, comparing, describing, questioning, discussing, investigating, reporting, collecting, selecting, testing, trying, listening, reading, drawing, calculating, imitating, modeling, playing, acting, taking on roles, talking, writing about what one can see, hear, feel, taste, experimenting and imagining.

Memorizing:

- Sequencing ordering, finding regularities and patterns, connect with given knowledge, use different modes of perception, depict.

Understanding:

- Structuring, ordering, classifying, constructing, solving, planning, predicting, transferring, applying knowledge, formulating ones individual understanding, interpreting, summarizing, evaluating, judging, explaining and teaching.

Organizing activities:

- The process of organizing activities must be based on curricular aims bringing together the needs, ideas, interests and characteristics of the children with the knowledge, skill, experience, and personality of the teacher within a given environment. The extent to which the teacher works with students individually or in groups affect the relation the teacher has with each child.

Steps required for Effective Organization of Activities.

a. Planning.

b. Involving children in the learning process.

c. Each child is made an active learner.

d. For each activity ensure you follow the principles of:-

- What?
- How? Work directions step by step, including:
- With whom? Where? How long?
- What after?

e. Ensure you give clear instructions before each activity. It must focus on the above a, b, c, d.

Role of a Teacher in an Activity Based Method

- A planner, an organizer and evaluator.
- Facilitator.
- Decision maker.
- Knowledge imparter
- Disciplinarian