Mathematical Propositions

Types and Proofs

Dr. Pooja
Assistant Professor

Statements or Propositions

- A proposition or statement is a declaration which is either true or false.
- Some examples:

- 2+2 = 5 is a statement because it is a false declaration.
- Orange juice contains vitamin C is a statement that is true.
- Open the door. This is not considered a statement since we cannot assign a true or false value to this sentence. It is a command, but not a statement or proposition.

Truth Table

- It is a table giving the truth values of a compound statement.
- It has a number of columns and rows.
- It is very helpful in finding out the validity of a report.

Compound Statement

It is a combination of two or more simple statements.

e.g. 1. Either he is an intelligent or he is hardworking.

2. $\sqrt{2}$ is real no. and $\sqrt{-2}$ is an imaginary no.

Connection Word	Name of Connective	Symbol
Or	Disjunction	V
And	Conjunction	Λ
If----then	Implication (conditional)	$\longrightarrow$
If and only of (iff)	Double implication (Equivalent)	$\longleftrightarrow$
Not	Negation	—

P is a true statement. ~P is false and Vice-Versa.

Example of Truth Table

p	q	r	$p \wedge q$	$\sim (p \wedge q)$	$\sim r$	$\sim (p \wedge q) \vee (\sim r)$
T	T	T	T	F	F	F
T	T	F	T	F	T	T
T	F	T	F	T	F	T
T	F	F	F	T	T	T.
F	T	T	F	T	F	T
F	T	F	F	T	T	T
F	F	T	F	T	F	T
F	F	F	F	T	T	T

Definition

- *Open Sentence* – an equation that contains one or more variables.
  - An open sentence is neither true nor false until the variable is filled in with a value.
- Examples:
  - Open sentence: $3x + 4 = 19$.
  - Not an open sentence: $3(5) + 4 = 19$.

Valid and Invalid Arguments

In mathematics and logic an *argument* is a sequence of statements ending in a conclusion. We now show how to determine whether an argument is valid—that is, whether the conclusion follows *necessarily* from the preceding statements. We will show that this determination depends only on the form of an argument, not on its content.

For example, the argument

If Jim is a man, then Jim is mortal.

Jim is a man.

$\therefore$ Jim is mortal.

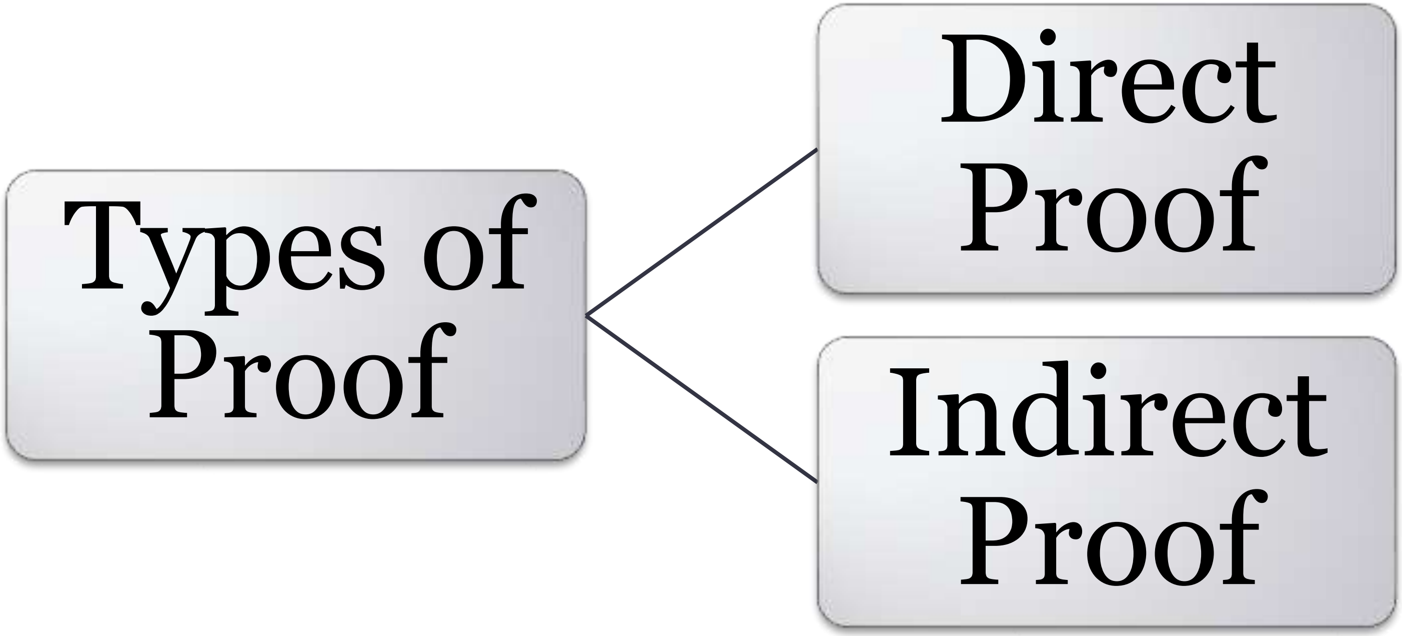
Converse, Inverse, and Contrapositive

- If you interchange the antecedent and the consequent of a conditional, you form a new conditional known as the ***converse*** of the original statement.
- If you negate both the antecedent and the consequent, you form the ***inverse*** of the statement.
- If you interchange and negate the antecedent and consequent, you form the ***contrapositive*** of the statement.

Related Conditional Statements		
Conditional (original)	$p \rightarrow q$	If p, then q.
Converse	$q \rightarrow p$	If q, then p.
Inverse	$\sim p \rightarrow \sim q$	If not p, then not q.
Contrapositive	$\sim q \rightarrow \sim p$	If not q, then not p.

Proofs

- A symbol of a theorem is finite sequence of logically valid steps that demonstrate that the premises of a theorem imply the conclusion.



Example of Direct Proof

- Theorem: The sum of two odd integers is even

1. let m and n be odd numbers,
   thus $n = 2s + 1$ and $m = 2t + 1$ — definition of odd number
2. $n + m = (2s+1) + (2t+1)$ — substitution
3. $= 2s + 2t + 1 + 1$ — associativity and commutatively
4. $= 2(s+ t + 1)$ — associativity and commutatively
5. n+m is even — definition of even

Indirect proof

- The method of proof *by contradiction* of a theorem $p \rightarrow q$ consists of the following steps:
    1. Assume p is true and q is false
    2. Show that ~p is also true.
    3. Then we have that p ^ (~p) is true.
    4. But this is impossible, since the statement p ^ (~p) is always false. There is a contradiction!
    5. So, q cannot be false and therefore it is true.
- OR: show that the *contrapositive* $(\sim q) \rightarrow (\sim p)$ is true.
    - Since $(\sim q) \rightarrow (\sim p)$ is logically equivalent to $p \rightarrow q$, then the theorem is proved.


$+ h \{a_n\}^k \quad \varphi \circ \cup$