Approaches on Teaching of Mathematics

2.4 METHODS OF TEACHING MATHEMATICS

We have seen that it is difficult to weave patterns with ideas which are essentially abstract. The elements of mathematics and the roles of intuition, logic, generalization and structure are all difficult areas to delimit or explore. What then, are proof structures or formats for organizing teaching-learning situations which a teacher can use? A few important lesson patterns are mentioned below.

2.4.1 Induction and Deduction

Mathematics in the making is experimental and inductive. Induction is that form of reasoning in which a general law is derived from a study of particular objects or specific processes. The child can use measurement, manipulatory or constructive activities, patterns, etc., to discover a relationship which he shall later formulate in symbolic form as a law or rule. The law, the rule or definition formulated by the child is the summation of all the particular or individual instances. In all induction the generalization that is evolved is regarded as a tentative conclusion.

Example 1: Ask pupils to draw a number of triangles. Ask them to measure the angles of each triangle and find their sum.

Conclusion: The sum of 3 angles of a triangle is 180° (approximately).

You can also ask children to cut the three corners of the triangles and put them at a point so that they form a straight line.

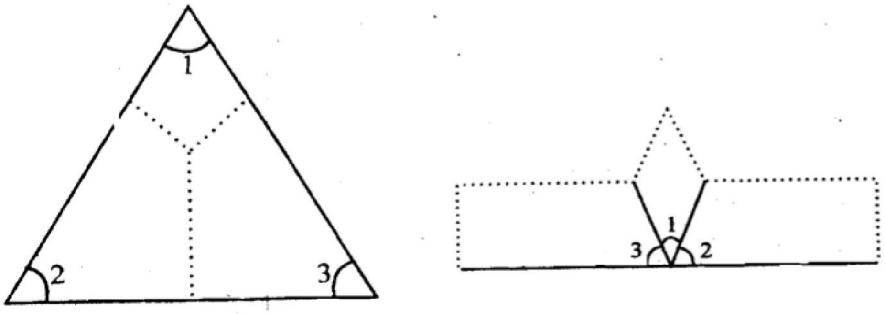


Fig. 2.1

Example 2:
$3 + 5 = 8$ sum of 2 odd numbers is an
$5 + 7 = 12$ even number
$9 + 11 = 20$

In deduction the law is accepted and then applied to a number of specific examples. The child does not discover laws but develops skills in applying them. We proceed from the general to the particular or from the abstract to the concrete. In actual practice the combination of induction and deduction is practised. The laws discovered by pupils inductively are further verified deductively through applications to new situations.

The difference of inductive and deductive method observed in teaching of mathematics is as follows:

Inductive Method	Deductive Method
1. Proceeds from the particular to the general; from the concrete to the abstract.	1. Proceeds from the general to the particular; from the abstract to the concrete.
2. It takes care of the needs and interests of children. It is a developmental prcess.	2. The child is provide with information of facts, principles & theories.
3. It encourages "discovery" and stimulates thinking.	3. It establishes linkage with real life observations and knowledge already gained.
4. The generalization or rule is formulated by the child therefore he remembers it with ease.	4. The rule is first learn and then derived by the child. So, he/she is likely to forget it.
5. The "how" and "why" of the rule/ generalization are made clear through reasoning.	5. The process is accepted by the child without much reasoning.
6. It starts from observation and direct experiences and ends in developing a rule in the abstract from.	6. It starts with a rule and provides for practice and applications.
7. It encourages child participation and group work.	7. It demands individual learning and treats a child as a passive recipient.

2.4.2 Analytic and Synthetic Methods

We have seen that in its early stages, most mathematics originates in ideas and concepts associated with physical form and shape. It is then presented as a systematic abstract structure in logico-deductive form. The ability to understand and work out a rigorous deductive structure using logic or reasoning is of great importance.

Analysis and synthesis are methods which use reasoning and arguments to discover relationships. Synthetic Euclidean geometry is a good model of a deductive structure and is favourable to the learning of reasoning and to the development of precision of thought. In any proposition we have (i) a hypothesis, which may be the information given in the proposition or a set of axioms, definitions, principles or relationships which have been proved earlier, and (ii) a conclusion, that is, the result to be proved or arrived at. Study the example given below.

Example: Prove that the sum of the three angles of a triangle is two right angles.

Here, the hypothesis is "a triangle (or the three angles of a triangle)" and the knowledge of result related to angles such as alternate angles, corresponding angles and the angle pairs which add up to two right angles (linear pair), etc., which are relations/definitions already proved prior to proving the given proposition.

The conclusion is "the sum of the three angles is two right angles".

In analysis we start from the conclusion and break it up into simpler arguments establishing connections with the relationships assumed in the hypothesis. In so doing, we find the missing logical connections and formulate a pattern for the proof. This pattern, when retraced from hypothesis to conclusion, gives the synthetic proof.

Split to simpler steps and establish logical connections
(Analysis)

Express the pattern in deductive form

(Synthesis)

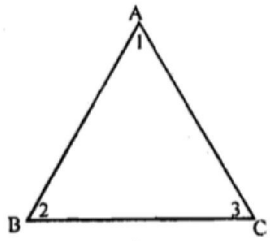


Fig. 2.2

Step 1: Let ABC be the triangle with angles 1,2, and 3.
To prove: $\angle 1 + \angle 2 + \angle 3 = 2$ right angles.

Step 2: We know that a linear pair measures 2 right angles.
Can we get a linear pair in fig. 2 ?

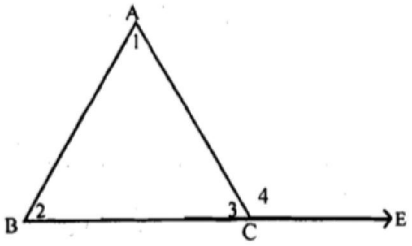


Fig. 2.3

Yes, but how? By producing the base BC to E.

Now $\angle 3$ and $\angle 4$ form a linear pair so

To prove: $\angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$

Step 3: Can we prove $\angle 1 + \angle 2 = \angle 4$?

We can at least cut an angle equal to $\angle 2$ from $\angle 4$ if we draw a line CD parallel to BA (Corresponding angles are equal).

$\therefore \angle 2 = \angle OCE$

Construction: Draw CD || BA

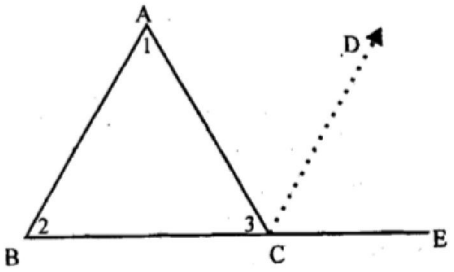


Fig. 2.4

Now can we show $\angle 1$ to be equal to $\angle ACD$?

Yes, This is true because $\angle 1$ and $\angle ACD$ are alternate angles. Hence the result. We can see how through arguments in step 2 and step 3 we arrive at constructions which help in developing the logical connections between the conclusion and the hypothesis.

The same sequence when written from the hyposthesis to the conclusion gives the synthetic proof.

Given: ABC is a triangle.

To prove: $\angle A + \angle B + \angle C = 2$ right angles.

Construciton: Draw CD || BA and extend BC to E (Fig. 2.4)

Proof: AB || CD (Const.) and BE meets them

$\therefore \angle ABC = \angle DCE$ (corresponding angles).....(i)

Again AB || CD and AC meets, them

$\therefore \angle BAC = \angle ACD$ (alternate angles) ..... (ii)

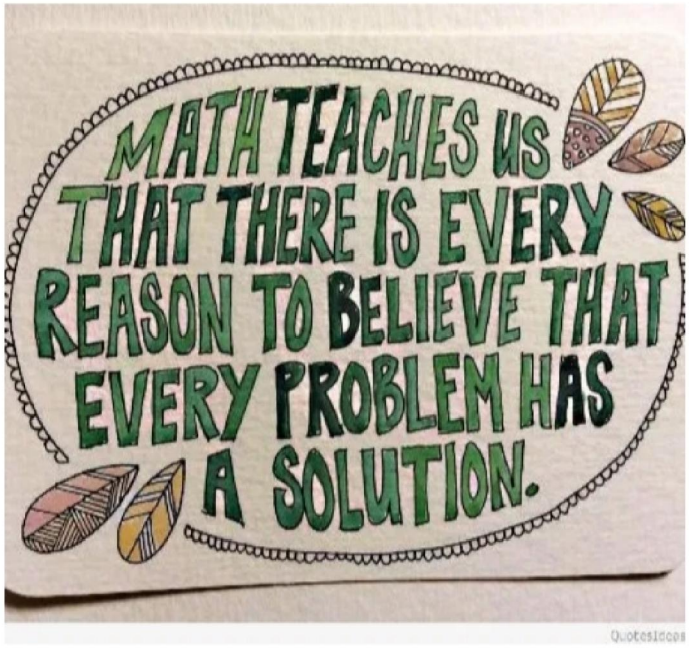
Adding (i) and (ii) : $\angle ABC + \angle BAC = \angle DCE + \angle ACD$

Add $\angle ACB$ to both the sides

$\angle ABC + \angle BAC + \angle BCA = \angle DCE + \angle ACD + \angle ACB = 2$ right angles.

Analytic Method	**Synthetic Method**
1. It proceeds from the conclusion to the hypothesis.	1. It proceeds from the hypothesis to the conclusion.
2. It involves breaking up the conclusion into simpler steps and setting up relationships with what is given or known. It applies intuition and inductive reasoning.	2. It involves writing out the steps in the proof in proper sequence using accepted deductive reasoning.
3. It is a method of discovery. The solution or proof is arrived at through systematic reasoning.	3. It is a method of presenting facts already discovered in a logical format.
4. It takes care of psychological considerations, self-learning, active participation of students, organized thinking and reasoning power. It builds up a scientific attitude, originality and creativity among the students.	4. It does not care for psychological principles. It is a logical method and encourages memorization of steps in proof.
5. The teacher acts as a guide and plans situations for discovery learning by students.	5. The teacher acts as a superior and explains the rationale of the proof.

PROBLEM SOLVING METHOD: METHODS OF TEACHING MATHEMATICS



PROBLEM SOLVING METHOD

Maths is a subject of problem. Its teaching learning process demands solving of innumerable problems.A problem is a sort of obstruction or difficulty which has to be overcome to reach the goal.

Problem solving is a set of events in which human beings was rules to achieve some goals – **Gagne**

Problem solving involves concept formation and discovery learning –**Ausube**

Steps in Problem Solving / Procedure for Problem solving

1. **Identifying and defining the problem:**

The student should be able to identify and clearly define the problem. The problem that has been identified should be interesting challenging and motivating for the students to participate in exploring.

2. **Analysing the problem:**

The problem should be carefully analysed as to what is given and what is to be find out. Given facts must be identified and expressed, if necessary in symbolic form.

3. Formulating tentative hypothesis

Formulating of hypothesis means preparation of a list of possible reasons of the occurrence of the problem. Formulating of hypothesis develops thinking and reasoning powers of the child. The focus at this stage is on hypothesizing – searching for the tentative solution to the problem.

4. **Testing the hypothesis:**

Appropriate methods should be selected to test the validity of the tentative hypothesis as a solution to the problem. If it is not proved to be the solution, the students are asked to formulate alternate hypothesis and proceed.

5. **Verifying of the result or checking the result:**

No conclusion should be accepted without being properly verified. At this step the students are asked to determine their results and substantiate the expected solution. The students should be able to make generalisations and apply it to their daily life.

Example :

Define union of two sets. If A={2,3,5}. B={3,5,6} And C={4,6,8,9}.

Prove that: AU(BUC)=(AUB)UC

Solution :

Step 1: Identifying and Defining the Problem

After selecting and understanding the problem the child will be able to define the problem in his own words that

1. The union of two sets A and B is the set, which contains all the members of a set A and all the members of a set B.
2. The union of two set A and B is express as 'AUB '

- The common elements are taken only once in the union of two sets

Step 2: Analysing the Problem

After defining the problem in his own words, the child will analyse the given problem that how the problem can be solved?

Step 3 : Formulating Tentative Hypothesis

After analysing the various aspects of the problem he will be able to make hypothesis that first of all he should calculate the union of sets B and C i.e. 'BUC' Then the union of set A and'BUC '. Thus he can get the value of AU(BUC) . Similarly he can solve (AUB)UC

Step 4: Testing Hypothesis

Thus on the basis of given data, the child will be able to solve the problem in the following manner

In the example it is given that

After solving the problem the child will analyse the result on the basis of given data and verify his hypothesis whether A U (B U C) is equals to (A U B) U C or not.

Step 5 : Verifying of the result

After testing and verifying his hypothesis the child will be able to conclude that

A U (B U C) = (A U B) U C

Thus the child generalises the results and apply his knowledge in new situations.

Merits

- This method is psychological and scientific in nature
- It helps in developing good study habits and reasoning powers.
- It helps to improve and apply knowledge and experience.

- This method stimulates thinking of the child
- It helps to develop the power of expression of the child.
- The child learns how to act in new situation.
- It develops group feeling while working together.
- Teachers become familiar with his pupils.
- It develops analytical, critical and generalization abilities of the child.
- This method helps in maintaining discipline in the class.

Demerits

- This is not suitable for lower classes
- There is lack of suitable books and references for children.
- It is not economical. It is wastage of time and energy.
- Teachers find it difficult to cover the prescribed syllabus.
- To follow this method talented teacher are required.
- There is always doubt of drawing wrong conclusions.
- Mental activities are more emphasized as compared to physical activities.

Conclusion

Problem solving is a suitable approach in teaching of mathematics. It develops in the learners the ability to recognize analysis, solve and reflect upon the problematic difficulties.

you can used all the methods discuss in my blogs as per the requirements. The twin combination of inductive deductive method and analytic synthetic methods are recommended as your day to day class. The inductive deductive method will be more suitable for arithmetic and algebra whereas analytic synthetic method will find greater application in plane geometry, trigonometry and solid geometry.

In some of the topics, it will be quite interesting to use project method or laboratory method. To budget the timing it will be good to use dogmatic method of teaching and for introducing new topic and reviewing topic lecture method with example can be more effective. At the end I can say that everyone have their own way of teaching and you can make your teaching more interesting by using combination of your own method and the method discuss in my blog.

Activity based approach of learning mathematics

Activity based learning focuses use of these sense organs and learning should be based on doing some hands-on experiments and activities. The idea of activity-based learning is rooted in the common notion that children are active learners rather than passive recipients of information.

The teaching-learning process is the heart of education
Teacher Subject Previously
Teacher Student Subject Now

The aim of activity-based approach is for learners to construct process of self-learning and problem solving and transfer of information and skills.

Activity Based Approach Child-centered educational aids to foster self learning and allows a child to study according to his/her aptitude and skill. Activities in each milestone include games, rhymes, drawing and songs to teach a letter or a word, or understand a concept. (Dhand, 1995) Need of Activity Based Approach Encourages independence and team learning Provides a wide variety of manipulative open- ended and creative activities Provides students experience and active participation Make students advance at their own rate (abilities, interest & motivations) Encourages self-reliance and development of initiative in an atmosphere of trust Encourages children to follow many of their own interests and desires to learn Need of Activity Based Approach Problem-solving, critical and creative thinking and deep understanding are emphasized Learners are encouraged to explore the new knowledge independently
Activity - I Sum of the three angles in a triangle is 180 degree
Activity - II Verification of Pythagoras Theorem The Square of the hypotenuse of a right angle triangle is equal to the sum of the square of the other two sides. Pre requisite Knowledge : Area of square, construction of parallel and perpendicular lines Material Required : chart paper, pen, glue stick, scissor.
. Procedure • Draw a right angle triangle on a chart paper. • Extend all the sides of triangle to form square • Draw unit squares in the squares corresponding two legs of triangle. • Cut the unit squares and paste them on square corresponding to hypotenuse. Area of square 1 = a*a = a2 Area of square 2 = b*b = b2 Area of square 3 = c2 = a2+b2 Pythagoras theorem : c2 = a2+b2
. Benefits ABL Approach • Children learn on their our pace. • Provision of more time for self – directed learning • Promoting Group, mutual and self learning• Teachers teaching time is judiciously distributed among children. • Children's participation is every step is ensured • Evaluation is inbuilt in the system
• Rote learning is discouraged • Periodical absence of child from school is properly addressed. • Classroom transaction is based on child's needs and interests. Benefits ABL Approach • Freedom to child in learning • Multigrade and multilevel in learning is effectively addressed. • No child can move to the next higher step of learning unless attain the previous one. • Sense of achievement boosts child's confidence and morale.
Attractive cards and activity create interest • Development in creative and communicative skill's. • Feel of security as they sit in round in the groups. Benefits ABL Approach • Allowed to move in the classroom • Distance between the teacher and the child is largely reduced • Teacher acts as a facilitator rather than teacher.
Conclusion "Mathematics learning should be imparted through activities from the very beginning of school education. i.e., form the primary stage itself. These activities may involve the use of concrete materials, models, charts, patterns, pictures, posters, games, puzzles, and experiments. The importance of using learning aids needs to be stressed." (NCERT, 2000)